Overvaluing 0-1 Bets

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Abstract

We document a new anomaly that standard preference models cannot capture, regardless of functional form or parameter specification used. Analyzing trading behavior in the binary option market for retail traders, we find that market participants purchase binary options although strictly dominant bull spreads are available at lower prices: 15% of S&P index, 19% of gold, and 25% of silver trades violate no-dominance conditions. Buyers of dominated binaries lose on average 34% of the contract price by forgoing dominating products. We prove theoretically that neither prospect theory nor ambiguity aversion can capture these results. We also test for, and reject, standard financial explanations including trading costs, liquidity, market maker concerns, and noise trader risk. The results are consistent with retail investors overvaluing easy-to-understand 0-1 bets. Our work provides one high-stakes, theoretically rigorous impetus for research in behavioral finance which goes beyond pervasive utility frameworks.

Keywords: retail investors, anomalies, behavioral finance

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1 Introduction

Theories of investor preferences, such as prospect theory and ambiguity aversion, have been tested extensively in the lab, but work in real-world, observational settings is more limited (Barberis, 2013). There are several reasons, which include that, first, most real-world financial settings have both retail and institutional capital. Since institutional capital is thought to be less subject to biases, its size and presence may hide retail investors' willingness-to-pay behavior, as noted in Odean (1998*b*) and Barber and Odean (2013). Second, most real-world settings feature both a supply and a demand side, and an extensive equilibrium literature shows that supply side dynamics alone, such as competitive concerns and hedging constraints, move prices in markets (Garleanu et al., 2009; Allen and Barbalau, 2022). However, preference theories model only the demand side (Kahneman and Tversky, 1979; Gilboa and Schmeidler, 1989).

In this paper, we find a clean, real-world setting, which serves as a well-identified, high-stakes environment in which to test preference models. In particular, we study a dedicated retail exchange that offers derivative contracts. The reason we can conduct a preference test in this market is that it is relatively small, oligopolistic, and explicitly targeted towards retail investors. As a result, traded prices are arguably more reflective of retail investors' willingness to pay than they would be in more competitive markets with a higher concentration of institutional capital. While these features may not be considered desirable were this an equilibrium asset pricing study, for the purpose of isolating retail willingness to pay as implied by preference models, they work in favor. At over $1 billion in notional value traded annually, our setting is also much larger and higher-stakes than any laboratory experiment, providing real-world tests of phenomena that are typically difficult to isolate and identify in observational data.

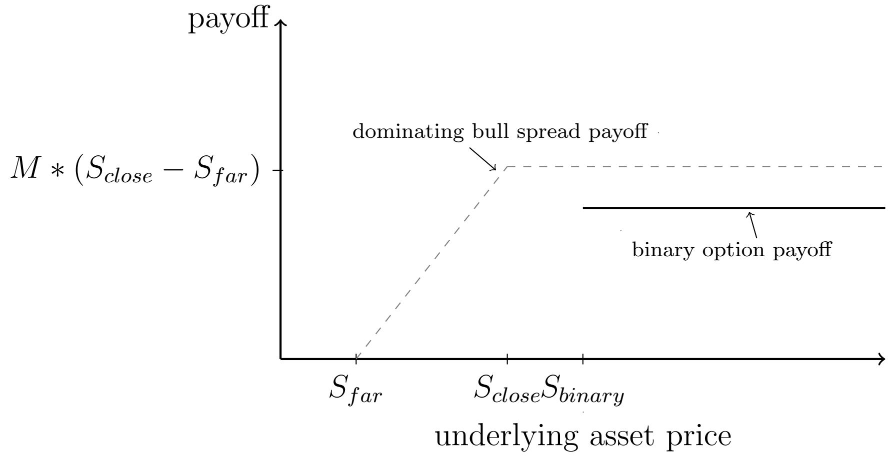
We document a new anomaly - overvaluing 0-1 bets - in this market. In addition, we go a step further by theoretically proving that this anomaly cannot be explained the most prominent preference models, regardless of functional form or parametric specification used.

Unlike past work that finds behavior not attributable to prospect theory (Bernheim and Sprenger, 2020), which is in the laboratory and has been subject to laboratory procedure critiques (Wakker, 2023; Bernheim and Sprenger, 2023), our setting is real-world and observational. In addition to informing investor behavior, our work therefore provides one high-stakes, theoretically rigorous impetus for a new wave of research in behavioral finance which goes beyond pervasive frameworks.

Our analysis focuses on state-by-state dominance comparisons. We use this tool, first and most importantly, to document a new anomaly, overvaluation of 0-1 bets. Second, we use this tool to test theoretical predictions, by deriving decision theoretic implications for observed behavior. We are able to do the latter because our setting has a relatively unexciting supply side, with only one market maker on one side of about 70% of trades (Commodity Futures Trading Commission Division of Market Oversight, 2017). As a result, we can attribute observed prices to retail willingness to pay, which can be difficult to do in larger and more competitive settings. To illustrate the difference that isolating demand makes, we observe that our results help resolve a tension in the literature. Whereas theoretical models of markets think of hard-to-understand products as delivering a premium on returns (Eisfeldt et al., 2023), empirical work finds that such products generate suboptimal returns (Allen and Barbalau, 2022). In our setting with price transparency, no repeated payments, and few supply side dynamics, we find overvaluation of easy-to-understand derivatives, indicating that the results in past empirical work may arise from supply side competitive considerations, for example product differentiation concerns, rather than from retail preference.

Our main result shows that traded prices for binary options - 0-1 bets - are higher than those of state-by-state dominating bull spreads, 15-20% of the time over the course of the year, regardless of which underlying asset one looks at. Buyers of these dominated binaries lose on average 34% of the contract price by forgoing the strictly better, non-binary product. This is not necessarily an arbitrage result, because to make money, one would have to take the other side of the trade, and bid-ask spreads on the retail platform are so

Figure 1: A Dominated 0-1 Bet



Note: We test whether the traded price of the dominated binary option is higher than the counterfactual price to purchase the dominating bull spread at the same point in time.

wide that this may not be profitable. We observe only traded prices in our data, but in our anecdotal experience with the platform, spreads as high as 30% or higher are not uncommon (we provide screenshots of the platform in Section 2). The message of our empirical result is only that buyers of binaries overvalue them: they could do strictly better by buying cheaper, dominating products.

Figure 1 summarizes the logic behind our empirical analysis. Because binary options give the holder a fixed payoff amount, a bull spread at nearby strike prices yields a payoff profile that is higher in every state of the world. We prove that any well-known theory of choice under risk – including the Nobel-Prize winning theories: expected utility, prospect theory and ambiguity aversion – predicts that willingness to pay for the strictly dominant bull spread should exceed willingness to pay for the binary option. The intuition is that both prospect theory and ambiguity aversion work by allowing for probability distortions, but because the bull spread pays more in every state of the world, no subjective probability distortion can make the binary the more appealing product. We prove this formally in the paper.

We further note that other behavioral theories, such as a preference for gambling cannot speak to the result that the dominated binary often trades at a higher price than the dominating bull. Both the bull spread and the binary represent gambles, and merely saying that someone likes to gamble does not justify why they systematically choose one gamble over the other. Skewness preferences, which is what a preference for gambling typically refers to, arises from prospect theory and would predict the opposite behavior, a selection of the bull spread over the binary, as we show.

In addition to preference theories, we also test for standard financial explanations for violations of the law of one price: random price volatility, liquidity premia, explicit trading fees, implicit trading costs arising from collateral requirements, trade size or market depth as reflecting leverage or liquidity constraints, and barriers to entry trading each product.

Which binary and bull spread data should we use? The binary option data is straightforward: for the identification reasons mentioned above, it comes from a dedicated retail binary exchange called Nadex. For bull spreads, we have two options. We could choose to use bull spreads on the same underlying which are *also* on Nadex, which immediately takes care of any knowledge or barriers to entry concerns, since both products are traded, and prominently featured, on the same platform. However, these bull spreads feature a higher price to enter (though possible net gains and losses are similar to binaries) and relatively few strike price options. Alternatively, we could take bull spreads from the highly liquid, both retail- and institutional-friendly Chicago Mercantile Exchange (CME), one of the world's largest exchanges for futures contracts. Barriers to entry to CME are similarly low, and there are many more strike prices than for the bull spreads on Nadex which makes a test resembling Figure 1 tighter and more powered. The prices for CME bull spreads are also closer to those of the binaries which helps take care of leverage and liquidity constraint concerns. We conduct tests for both kinds of bull spreads - on- and off-platform - and show that the binary is over-priced regardless of which bull spread type we look at. In the main paper, to match more trades and for a cleaner test, we focus on the CME bull spreads. Section 5.4 re-conducts this analysis for the bull spreads within Nadex, and shows that the dominance violations we document continue to hold within-platform. As a result, exchange-specific fixed effects cannot explain our results.

We now describe our tests for standard financial explanations for anomalies in greater detail. To assess the role of random price volatility, we compute overvaluation anomaly frequency when the binary option is dominating, rather than dominated. If random price noise drives the results, then it should operate equally in both directions, meaning that overvaluation anomaly frequency should be similar when the binary option is the better product. We show that this is not the case, as anomaly frequency is significantly smaller – both statistically and numerically – when the binary option is dominating, implying that random price noise cannot explain our results.

Regarding fees, our price comparisons control for all explicit trading costs. Differences between the binary and bull in collateral requirements or market liquidity might create implicit trading costs, but we can reject these possibilities as well, both because of institutional detail as well as through a statistical test. A collateral-based explanation would posit that prices are higher for binaries if the bull spread requires collateral but the binary does not. In practice, however, neither requires collateral to buy since no downside losses are involved. For completeness, we nevertheless test whether overvaluation anomaly frequency is higher during periods of higher market volatility (when collateral at bull spread brokers may be more likely to be requested). The relationship between market volatility and anomaly frequency is too weak to play a substantial role in explaining our results.

Liquidity premia cannot account for our findings, either, since investors should be willing to pay a premium to trade in the more liquid CME market. Instead, we find that dominating bull spreads often cost *less* than dominated Nadex binary options.

We also consider the possibility that retail investors are asymmetrically informed about binaries versus spreads. However, Nadex offers both binaries and call spread variants prominently on its platform, with educational and intuitive explanations provided for each. As we document through screenshots, both products can be easily traded and are prominently displayed, at the top of the screen. Nevertheless, within Nadex, we find that binaries are over-valued 15.1% of the time.

Our results document a novel anomaly - overvaluing 0-1 bets - in new observational data. They also suggest an impetus for expanding beyond Nobel-winning utility models, and provide a data point for future theoretical and empirical work on investor behavior.

Part of the message of our empirical paper is a call to action to expand beyond well-accepted behavioral theories. We are open to a number of explanations - cognitive, preference-based, etc. - and have no strong stance on what drives our results. Our message is only that they occur and that the most popular theories cannot explain them. One possibly interesting explanation future theoretical work could explore further is that binary options are easier to understand, in many ways, than bull spreads. Binaries are 0-1 bets that require no construction. Bull spreads (on CME) require construction and have many possible outcomes. Bull spreads within same exchange as binaries (Nadex) also require no construction but may be harder for a retail trader to conceptualize, e.g. in terms of expected value or payoff structure, than a 0-1 bet.

We contribute to understanding of retail investors' behavior, and in particular to a literature documenting anomalies. Closely related papers in this literature include the anomaly of the disposition effect (Odean, 1998*a*; Barber and Odean, 2004; Barber et al., 2009*a*,*b*), sentiment and emotion (Tetlock, 2007; Kuhnen and Knutson, 2011; Kuhnen and Melzer, 2018) including their effects on options trading behavior (Han, 2008), and overconfidence (Odean, 1999; Barber and Odean, 2000; Gervais and Odean, 2001), as well as anomalies in derivatives markets more broadly (McDonald, 2013; Cheng, 2019).

The paper proceeds as follows. Section 2 provides an overview of the binary option market. Section 3 details our empirical strategy, including our trade-matching algorithm. Section 4 provides the results. Section 5 performs robustness tests as described above, including within-Nadex analysis to control for exchange fixed effects. Section 6 concludes.

The appendix contains additional supplemental tests that complement the main text, but are not critical for our results. These include a survey of binary option traders to elucidate demographics and trading behavior, as well as Black-Scholes-Merton analysis of binary options.

2 Institutional Detail

Despite their short history among retail traders, binary options have generated considerable controversy. Since their introduction to retail exchanges in 2008 (SEC, 2008), these derivatives have been temporarily or permanently banned by multiple countries including the European Securities and Markets Authority (ESMA, 2019), Israel (Weinglass, 2019), Denmark (Finanstilsynet, 2019), and the United Kingdom (FCA, 2019), amidst concerns that retail investors have lost too much money trading binary options.

In the U.S, however, the binary option market is both legal and growing. The largest binary option exchange in the US is called Nadex. Founded in 2007, Nadex offers binary options, call spreads, and knock-outs to its clients. Our focus is on the binary options and call spreads. Figure 2 shows sample bid and ask spreads on the platform, showing that both binary options and call spreads are offered prominently on the top of the screen. For a given selected product, the left-most side of the screen shows these bids and asks, the center a visual depiction of payoffs, and the right side maximum profit and loss from entering the trade.[1] These same aids - visual depictions and maximum profit and loss - are also displayed for call spreads.

Table 1 shows the percent of trades in each type of contract. One immediately notices an outsize fraction of trades in binary options (94%) relative to call spreads (5.2%). This is consistent with retail investors being drawn to binaries, but in itself is not a clean well-identified test, and does not rule out either behavioral or rational theories. Our analysis will go further than descriptive statistics on trade volume.

[1]By default, these are measured assuming a market order, and some possibility of the market ordering executing at a worse price than that offered by the bid-ask spread.

(A) Weekly Binary Options

(B) Daily Binary Options

FIGURE 2: Sample Bid-Ask Spreads on Nadex

Note: This figure shows sample bid and ask prices for weekly and daily binary option on S&P 500 futures, as well as daily call spreads on S&P 500 futures. Observe that both binary options and call spreads are offered prominently on the top of the screen.

Instrument Type	Percent of Trading Volume
Binary Option	94.1%
Spread Option	5.2%
Bracket Option	0.7%

TABLE 1: Product Breakdown on Nadex

Nadex provides a high-stakes setting in which to study revealed preference. During our year of data between May 2018 and May 2019, Nadex executed over 4.2 million binary option trades. The aggregate market value of these trades (using trade prices at the time of execution) was over $513 million, and the aggregate notional value (with all Nadex binary options giving a $100 fixed payoff) was just over $1.04 billion. As observed in the introduction, these real-world transactions are much higher stakes - at over 1 billion dollars - than would be found in any laboratory experiment. At the same time, the retail focus and relative lack of institutional capital, makes the identification of retail-driven behavior in this market cleaner than in a larger and more competitive setting.

The binary option market itself has widened recently, with large institutional players entering in the belief that retail investors are excessively drawn to these contracts. For example, crypto.com acquired Nadex in 2023, and the CME Group (one of the world's largest derivatives exchanges) launched binary option contracts in September 2022, with the express goal of attracting more retail investors (Platt and Rennison, 2022; Crypto.com, 2022). Appendix B.2 re-conducts our analysis for the year post expansion and shows that the frequency of price anomalies decreases after these large institutional players enter, consistent with supply-side effects and institutional capital differing from retail demand-side dynamics.[2] This further validates our setting and period - prior to the expansion of the binary market - as well suited to studying revealed preference in a high stakes, non-laboratory setting.

3 Identification Strategy

3.1 Test

Figure 1 illustrates the logic behind our empirical test for over-valuing binaries. A binary option with a strike price of S_{binary} pays 0 to the buyer if the underlying asset price is

[2] Célérier and Vallée (2017) has analysis of complexity premia in more competitive settings. As mentioned in the introduction, these results are the opposite of what we find in our price-transparent retail market.

below S_{binary} at contract expiration, and pays some fixed positive amount if the underlying asset price finishes above S_{binary}. Graphically, this payoff profile is the solid horizontal line extending rightward from S_{binary}.

Unlike the fixed payoff of a binary option, a call option that settles in the money pays the buyer the difference between the underlying asset price and the contract's strike price. Buying a call option at strike price S_1 thus creates a payoff profile extending rightward from S_1 with a slope of 1, as shown by the dashed line in Figure 1. If the difference between the strike prices S_1 and S_2 is chosen to match the binary option's fixed payoff amount, this upward-sloping dashed line intersects with the solid binary option payoff exactly at S_2. If the buyer of the call option with strike price S_1 also sells a call option with strike price S_2, the losses from the latter exactly cancel the winnings from the former as the underlying asset price moves above S_2. The net payoff profile of this two-contract portfolio (buying a call at S_1 and selling a call at S_2) thus levels off and becomes higher than the horizontal binary option payoff for all underlying asset prices above S_2.

Comparing the payoff profiles in Figure 1 shows that the payoff profile of the bull spread strictly dominates the payoff profile of the binary option, state-by-state.[3] The payoffs are identical for all underlying asset prices below S_1, but the portfolio of call options pays off a strictly higher amount than the binary option for all underlying asset prices between S_1 and S_{binary}, as well as above S_{binary}.

Any buyer whose preferences respect dominance must therefore be willing to pay more for the bull spread than the binary option. As we prove in Section 5, even theories that do not always satisfy dominance - prospect theory and ambiguity aversion - predict a higher willingness to pay for the bull spread than the binary. As a result, our theoretically-motivated test for an overvaluation of binary options is simply whether the traded price on the dominated binary is strictly higher than the price the buyer could have paid for a dominating

[3]Dominance comparisons have been used as tools for analysis in other settings (Jensen and Pedersen, 2016; Bhargava et al., 2017; Egan, 2019; Vokata, 2021); here, we use this tool to document a new anomaly.

bull spread at the exact same point in time. The mathematical formulation is:

$$\text{dominance satisfying} \implies P_{S_1} - P_{S_2} > B_{S_{binary}}. \tag{1}$$

A violation of this condition implies that the buyer of the binary has overpaid for it.

3.2 Data

Our binary option data comes from Nadex. As discussed in the Introduction, data on bull spreads can come from two sources: Nadex (its call spreads) or the more liquid, highly competitive CME Globex market. We focus on the latter for its depth, more similar contract size, and ability to match quotes closely in time to the traded binaries. As a robustness check to account for exchange fixed effects, we also conduct analysis within-Nadex and show in Section 5.4 that the over-valuation results we find continue to hold.

We focus on three underlyings: the S&P 500 index, gold, and silver. The underlying asset for S&P index options is the near-month CME E-Mini S&P 500 Index futures price, and for gold and silver options, it is the near-month COMEX futures price. We focus on these three underlyings because they have comparable bull spreads at CME.

In the Nadex data, we observe all trades in weekly and Friday daily contracts for the S&P, gold, and silver markets for one full year between May 17, 2018 and May 17, 2019. For each observed trade, the Nadex data show the execution time, the strike price, the transaction price, and the number of contracts traded. The payoff profile for buyers of these options is exactly as shown in Figure 1, with the fixed payoff amount equal to \$100 for all contracts. There is a \$1 trading fee per contract, as well as a \$1 settlement fee per contract for positions that expire in the money. We factor these fees into our calculations when comparing Nadex transaction prices to CME price quotes.

The CME weekly options we use expire on Friday afternoon[4] and settle to the same

[4]Note that while both the Nadex binary options and the CME call options we study expire on Friday afternoon, they do not expire at exactly the same time. Nadex's weekly and Friday daily contracts in

underlying asset price (near-month CME E-Mini S&P 500 Index futures price for S&P index options, near-month COMEX futures price for gold and silver options) as the Nadex binary options described above.[5] We obtain top-of-book data from CME for the same yearlong sample period between May 17, 2018 and May 17, 2019. The CME data show changes in the top-of-book best bid and ask price quotes as well as executed trades. For each price quote or executed trade, we observe a time stamp, the strike price, the quoted or transaction price, and the number of contracts listed or traded. The side of the market (i.e., ask vs. bid) is indicated for changes in top-of-book price quotes, but not for executed trades. To price the dominating bull spread, which traders may purchase directly, we follow standard brokerages by pricing the bull as equivalent to buying a call option at one strike price and selling it at the next (TD Ameritrade, 2021). If these call options settle out of the money, buyers receive nothing; if they settle in the money, buyers receive the difference between the underlying asset price and the strike price, multiplied by the notional amount of the contract. The notional amounts of the S&P, gold, and silver options are $50, 100 troy ounces, and 5,000 troy ounces, respectively. CME charged a trading fee of $0.55 per contract for weekly S&P options and $1.45 per contract for weekly gold and silver options during our sample period, which we factor into our calculations when computing price differences.[6]

gold expire at 1:30pm ET, and those in silver expire at 1:25pm ET. CME's weekly gold and silver options both expire at 5:00pm ET. Since options that expire later are more valuable, this difference in expiration times only makes the CME portfolios we construct more appealing relative to Nadex binary options, and thus cannot explain price anomalies where dominating bull spreads in gold and silver cost less than their corresponding binary options. As for the S&P index options, Nadex's weekly and Friday daily contracts expire at 4:15pm ET while CME's weekly options expire at 4:00pm ET. The difference in expiration times for the S&P index options thus works the other way and would tend to make Nadex binary options more valuable (all else equal) than corresponding CME portfolios. However, we expect any valuation difference arising from this 15-minute discrepancy to be small, and incapable of explaining the frequent and large price anomalies we document in Section 4.

[5]The CME exchange tickers for the S&P, gold, and silver options we use begin with "EW," "OG,", and "SO," respectively.

[6]We use the trading fees charged to non-members. This is a conservative assumption because fees charged to non-members are higher than fees charged to CME members, and we seek to find cases where buying the dominating bull spreads costs less than the traded price of the binary option.

3.3 Matching Algorithm

We carry out our empirical strategy by matching observed binary option trades to call price quotes and trades around the same time, then computing price differences to test for violations of the dominance condition (1). In particular, for each binary option trade, we perform the following analysis:

1. Subset call option quotes and trades to those that occurred in the 10 minutes before the binary trade. We do this in order to identify the bull spread prices (and any resulting price anomalies) that the buyer of the binary option passed up in the minutes before executing his or her trade.

2. Further subset the quotes and trades to those at the two highest strike prices that are weakly less than the binary option trade's strike price. Formally, denoting S_{binary} as the binary trade's strike price and $\{S_i\}_{i=1}^{N}$ as the N unique strike prices among the call option quotes and trades, define:

$$S_{close} = \max\{S_i | S_i \leq S_{binary}\}, \tag{2}$$

$$S_{far} = \max\{S_i | S_i < S_{close}\}. \tag{3}$$

3. For both strike prices S_{close} and S_{far}, choose one quote or trade price to use in evaluating the dominance condition. Do so by using the following priority ordering:

   - First, prioritize price quotes that are on the correct side of the market, assuming that the trades used to construct our dominating bull spread would be taking liquidity. In other words, since our bull spread is constructed by buying at S_{far} and selling at S_{close}, prioritize quotes at S_{far} that are on the ask side of the market, and quotes at S_{close} that are on the bid side of the market. Note that the call option data indicates market side for price quotes but not for executed trades; we thus use executed trade prices only if we cannot find an appropriate price quote in our 10-minute window. We flag instances where we are forced to use a quote from the wrong side of the market or an executed trade price and exclude them in one of our robustness checks.

- Second, prioritize quotes and trades that occurred closer in time to the binary option trade.

4. Then compute the difference between the binary trade price and the price of the dominating bull spread, using the quote or trade prices chosen in the above steps.

   - The price of the bull spread is

$$P_{S_{close}} - P_{S_{far}} + 2F_{CME}, \tag{4}$$

   where P_i is the price of the call option with strike price i and F_{CME} is the CME trading fee per contract ($0.55 for S&P options and $1.45 for gold and silver options).

   - The difference in strike prices $S_{far} - S_{close}$ determines the height at which the bull spread payoff profile levels off into a horizontal line (see Figure 1 below). Since the fixed payoff amount of binary options is always $100 (minus the Nadex settlement fee that must be paid if the contract settles in the money), we must scale the binary option trade quantity to ensure that the binary and the bull payoff profiles are the same at all underlying asset prices above S_{binary}. The price of the scaled binary trade is thus

$$\frac{M * (S_{close} - S_{far})}{100 - F_{settle}} * \left(B_{S_{binary}} + F_{trade}\right), \tag{5}$$

   where M is the notional amount of the CME contract ($50 for S&P options, 100 troy ounces for gold, and 5,000 troy ounces for silver), B_i is the price of the binary option with strike price i, F_{settle} is the Nadex settlement fee of $1 per contract, and F_{trade} is the Nadex trading fee of \$1 per contract.

- Evaluating the dominance condition thus reduces to

$$M * (P_{S_{far}} - P_{S_{close}}) + 2F_{CME} \lesseqgtr \frac{M * (S_{close} - S_{far})}{100 - F_{settle}} * \left(B_{S_{binary}} + F_{trade}\right). \quad (6)$$

If the left-hand side of expression (6) is smaller, we conclude that the dominating bull spread costs less than the scaled binary trade and the dominance condition is violated. Note that the scale factor $\frac{M*(S_{far}-S_{close})}{100-F_{settle}}$ is generally not an integer, and trading a fractional number of binary contracts is not possible under exchange regulations. We thus adjust fractional scale factors down to the closest possible integer scale factor that preserves the bull spread's dominating status. Note also that the observed trade quantity in the binary option data is usually smaller than the scale factor, so we are implicitly assuming that the binary option trader would have been willing to scale up his or her trade and purchase $\frac{M*(S_{far}-S_{close})}{100-F_{settle}}$ contracts. We do not have to make this assumption when the observed trade quantity is actually greater than or equal to our scale factor, so a final robustness check below limits to these cases.

4 Results

4.1 Summary Statistics

Table 2 presents summary statistics for our sample of binary option trades. Together, the S&P, gold, and silver binary options provide a sample of 55,008 total trades. Of these, 25,621 are weekly contracts while the remaining 29,387 are Friday daily contracts. Nadex traders during our sample period put moderate amounts of money at risk: the average trade price for all three option types is around \$50 per contract, and the average sizes of S&P, gold, and silver trades are 4.24, 3.77, and 2.10 contracts, respectively. Because the CME top-of-book data contain millions of price quotes and trades per day, our matching algorithm is successful most of the time, as we match 87% of S&P trades, 93% of gold trades, and 88% of silver trades to a dominating bull spread.[7]

Aside from showing the overall success rate in matching binaries to dominating bull spreads, Table 2 also summarizes how "close" the binaries are to their matched bull spreads, in terms of both time and strike price placement. The average time difference between a binary option trade and the price quotes or trades in its matched bull spread is about 25 seconds for S&P options, about 30 seconds for gold options, and about 55 seconds for silver options. Our 10-minute cutoff for locating bull spread prices is thus rarely binding, and tightening this cutoff does not noticeably change the results. The matched bull spreads also contain strike prices that on average are close to the binary strike price. However, the standard deviations of these strike price differences are fairly large, and Online Appendix B.1 disaggregates the anomaly results based on these distances (with larger strike price distances indicating larger or more severe dominance violations). Finally, Table 2 summarizes the scale factor that we use to match the payoff profiles of binary options and their corresponding bull spreads. As discussed in Section 3.3, to ensure that the payoff profiles of the binary and bull spread level off at the same height, we must purchase $\frac{M*(S_{close}-S_{far})}{100-F_{settle}}$ (rounded down to the nearest integer) binary options. The average scale factor is 2.03, 5.15, and 2.80 for S&P, gold, and silver options, respectively.

4.2 Overvaluing 0-1 Bets

Our matching algorithm produces, for each matched binary, the price of a dominating bull spread that was available just before the time of the binary trade. Cases where the

[7]CME does not list weekly S&P options with expiry dates on the third Friday of March, June, September, or December (because their quarterly S&P options expire on these dates). Most (69%) of the unmatched S&P trades are for contracts that expire on the third Friday of March, June, September, or December, and thus have no concurrent CME trading activity. Another 7% of the unmatched S&P trades occur on Fridays between 4:10pm and 4:15pm and thus have no CME trading activity within our 10-minute window (since CME S&P options expire on Fridays at 4:00pm ET; see again footnote 4). The rest of the unmatched S&P trades, and all of the unmatched gold and silver trades, are dispersed evenly throughout the sample period, mostly in the early-morning or late-evening hours when CME trading volume is relatively thin.

Table 2: Summary Statistics for Binary Option Trades

	S&P	Gold	Silver
Trade Price	51.28	46.11	46.37
	(24.07)	(24.74)	(26.94)
Quantity	4.24	3.77	2.10
	(13.88)	(10.45)	(3.28)
Matched to Bull Spread	0.87	0.93	0.88
	(0.34)	(0.25)	(0.33)
Time Diff. to Close Strike (secs)	22.76	32.79	57.41
	(76.93)	(76.74)	(105.06)
Time Diff. to Far Strike (secs)	25.30	25.54	55.32
	(81.23)	(64.15)	(105.72)
Time Btw. Close/Far Strikes (secs)	12.63	27.10	29.94
	(45.26)	(67.49)	(79.22)
Close Strike Distance	2.19	2.53	0.01
	(1.67)	(7.39)	(0.05)
Far Strike Distance	7.25	7.68	0.08
	(2.16)	(8.83)	(0.11)
Scale Factor	2.03	5.15	2.80
	(0.53)	(4.85)	(4.14)
Weekly Trades	19,691	4,875	1,055
Friday Daily Trades	25,420	3,101	866
Total Trades	45,111	7,976	1,921

Note: All variable names are self-explanatory except those concerning close and far strike prices and the scale factor. See Section 3.3 and Figure 1 for definitions of the close and far strike prices in the matched bull spread. The distance variables here give the difference between the binary strike price and the close/far bull spread strike prices (in index points for the S&P options, in dollars per troy ounce for the gold and silver options). The scale factor is the number of binary contracts that must be purchased in order to match the height of the bull spread's payoff profile; again see Section 3.3 and Figure 1 for discussion of the scale factor.

dominating bull spread costs less than the (appropriately scaled) binary option constitute anomalies. Figure 3 summarizes the distribution of these price differences, in both absolute dollar terms (bull spread price minus the scaled binary price) and relative terms (the absolute difference divided by the price of the scaled binary trade). In each case there is substantial mass to the left of zero, indicating that violations of the no-dominance condition arise often.

The main overvaluation anomaly results are presented in Table 3. The baseline results, which consider all binary trades that were successfully matched to a dominating bull spread, show that anomalies occur often. We find that 15% of the 39,283 matched S&P options, 19% of the 7,434 matched gold options, and 25% of the 1,681 matched silver options cost more than purchasing their dominating bull spread would.[8]

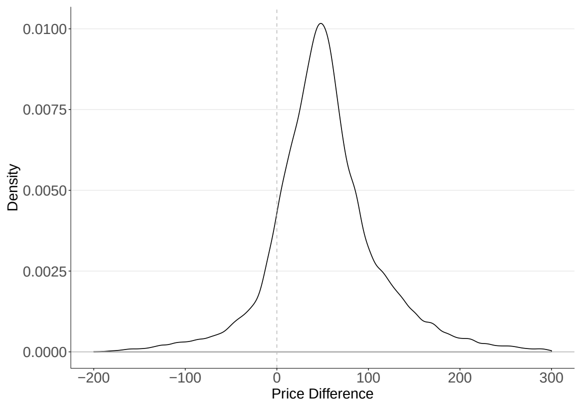
Furthermore, conditional on an overvaluation occurring, the price differences between dominated binary options and dominating bull spreads are large. Because the distributions of price differences are generally right-skewed, we report means as well as medians. In the baseline sample the mean price differences for S&P, gold, and silver trades are $40.41, $44.51, and $28.68, while the median price differences are $25.40, $29.60, and $20.60, respectively. In relative terms, the price difference represents an average of 34%, 25%, and 35% of the binary option trade price for the S&P, gold, and silver asset classes, with the median figures slightly smaller.

Table 4 shows four robustness checks that were introduced during the discussion of the matching algorithm in Section 3.3. Columns 1-3 restrict to cases where we are able to find price quotes on the correct side of the market – i.e., cases where the price quote at the far strike price comes from the ask side of the market and the price quote at the close strike price comes from the bid side of the market. Since we are almost always able to find price quotes on the correct side of the market, the sample sizes only decrease by a small amount and the results are largely unchanged. Columns 4-6, which drop daily binary options and consider

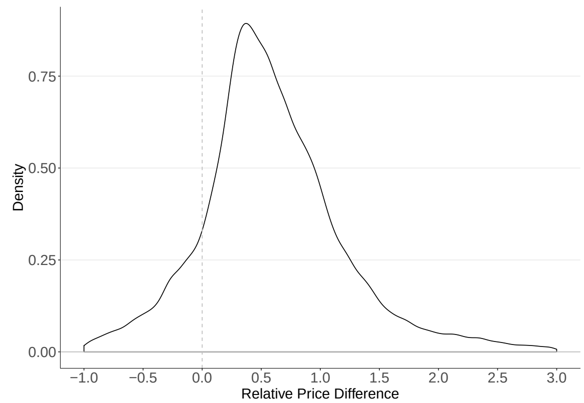
[8]For a small number of binaries, the matched call option at the far strike price actually costs less than the matched call option at the close strike price (see again Figure 1). We still count these cases as price anomalies when computing anomaly frequency, but conservatively exclude them when computing conditional overvaluation anomaly sizes (since the overvaluation anomaly sizes for these trades are often large outliers).

Figure 3: Price Differences Between Dominating Bull Spreads and Dominated Binaries

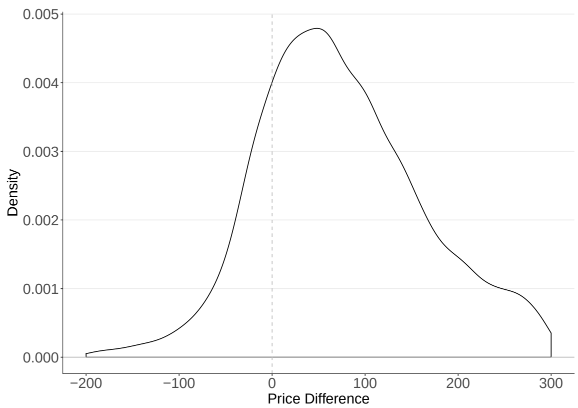
(a) S&P: Absolute Price Differences



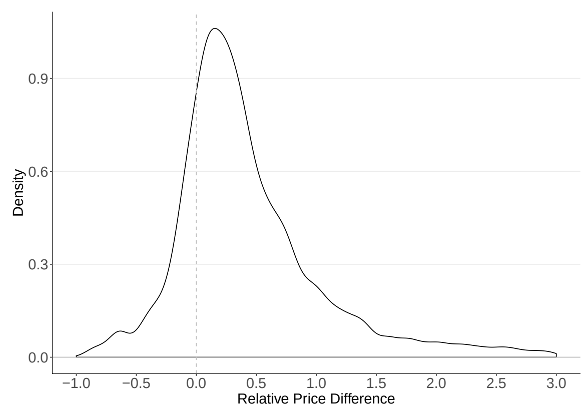
(b) S&P: Relative Price Differences



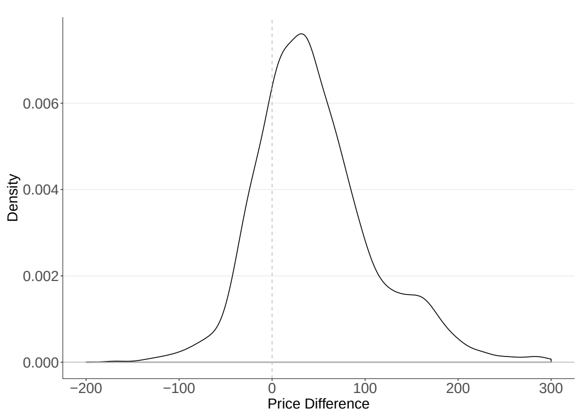
(c) Gold: Absolute Price Differences



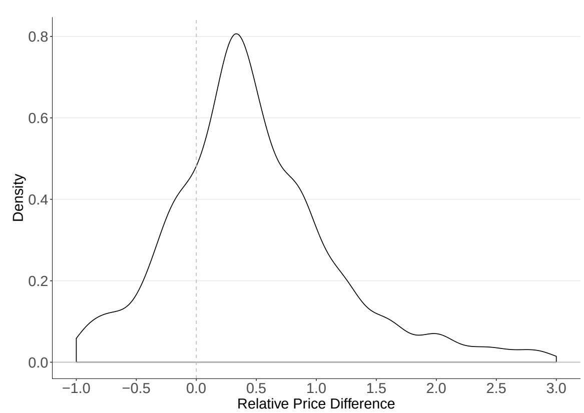
(d) Gold: Relative Price Differences



(e) Silver: Absolute Price Differences



(f) Silver: Relative Price Differences



Note: These graphs summarize the price differences between binary options and their matched bull spreads. The plotted variable is the price of purchasing the matched bull spread minus the price of the binary option; negative values thus indicate overvaluation anomalies. The absolute price difference is the raw dollar value, and the relative price difference divides the absolute price difference by the price of the scaled binary trade (i.e., the binary trade price times the scale factor).

Table 3: Overvaluation of Binaries: Baseline Results

	S&P	Gold	Silver
Overvaluation Frequency	0.15	0.19	0.25
Overvaluation Size			
Mean	40.41	44.51	28.68
Median	25.40	29.60	20.60
Overvaluation Size/Price			
Mean	0.34	0.25	0.35
Median	0.27	0.15	0.27
Binary Option Trades Considered	39,283	7,434	1,681

Table 4: Overvaluation of Binaries: Robustness Tests

	Prices on Correct Side			Weekly Trades Only			Scale Factor ≤ Quantity			Fractional Scale Factors		
	S&P (1)	Gold (2)	Silver (3)	S&P (4)	Gold (5)	Silver (6)	S&P (7)	Gold (8)	Silver (9)	S&P (10)	Gold (11)	Silver (12)
Overvaluation Frequency	0.14	0.18	0.24	0.17	0.23	0.32	0.15	0.19	0.26	0.28	0.20	0.39
Overvaluation Size												
Mean	38.52	43.09	29.26	36.72	39.94	27.81	38.50	40.39	25.95	40.87	44.93	35.87
Median	25.40	28.35	21.60	22.40	25.85	20.60	24.90	25.85	18.60	21.68	29.42	23.49
Overvaluation Size/Price												
Mean	0.33	0.21	0.33	0.34	0.24	0.37	0.34	0.22	0.34	0.28	0.24	0.33
Median	0.26	0.14	0.26	0.28	0.14	0.28	0.26	0.12	0.29	0.20	0.15	0.30
Binary Option Trades Considered	38,806	7,221	1,591	17,158	4,542	939	15,095	985	470	39,283	7,434	1,681

Note (to Tables 3-4): Only binary options that were successfully matched to a dominating bull spread are considered. The overvaluation anomaly frequency is the fraction of the relevant binary options that cost more to buy than their dominating bull spread. Overvaluation size is the difference between the scaled binary trade price (i.e., the binary trade price times the scale factor) and the cost of the dominating bull spread, conditional on the binary option costing more and an overvaluation anomaly existing. Overvaluation size/price normalizes overvaluation size relative to the price of the scaled binary trade.

only weekly binary options, see overvaluation anomaly rates increase modestly. Columns 7-9 limit to trades where the observed trade quantity in the binary data is at least as large as the scale factor: in these cases, our assumption that the binary trader would have been willing to purchase the scaled-up number of binary contracts is obviously true. The sample sizes decrease significantly but the results are again essentially unchanged relative to the baseline. Finally, columns 10-12 allow fractional scale factors. Relaxing the constraint on integer scale factors increases overvaluation rates but does not have much of an effect on conditional overvaluation size.

5 Assessing Possible Explanations

In addition to the four robustness checks of Section 4, in this section we consider standard explanations for anomalies and show that they are unable to capture overvaluation of 0-1 bets.

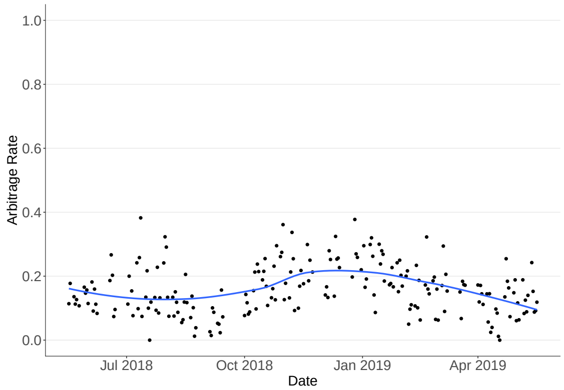
The financial explanations we consider and reject are random price noise; implicit or explicit trading costs; liquidity premia; differential barriers to access; risks to execution; trade size or market depth; and constraints on the market maker side.

The behavioral explanations we prove are theoretically incompatible with these results are prospect theory, ambiguity aversion, rational inattention, as well as more recent developments such as salience, reference dependence, and disappointment aversion. In the final part of this section, we note that the results could be consistent with a premium on the easier-to-understand binary.

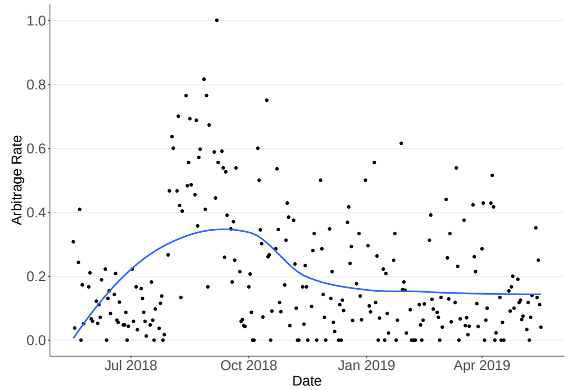
5.1 Random Price Noise

We start by considering a random price noise explanation. If particular market events or short periods of low liquidity for binaries caused price volatility that in turn temporarily moved prices outside of no-dominance bounds, then we might expect to see spikes in overvaluation frequency during particular parts of our sample period. Similarly, if market thinness during particular parts of the trading day caused excess price volatility, we would expect to see increases in anomaly frequency rates during certain trading hours.

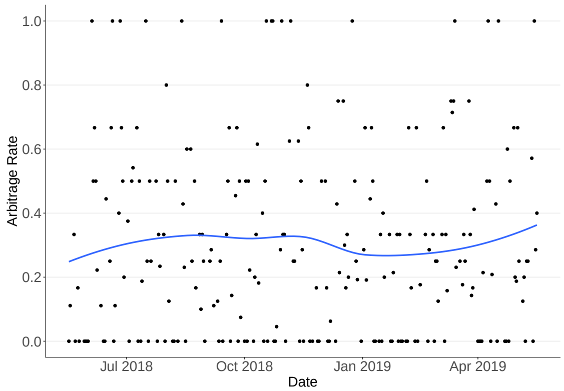
Figures 4a-4d investigate these possibilities. In Figures 4a-4c, we plot the daily overvaluation frequency (which, as defined in Section 3, is the fraction of binary trades that cost more than their dominating bull spread), along with a smoothed local linear trend, for all three option types. Gold overvaluation frequencies rates are noticeably higher in the second half of 2018, but remain around the average value of 19% throughout the sample period; S&P and silver overvaluation frequencies are roughly constant over time. Daily silver overvaluation



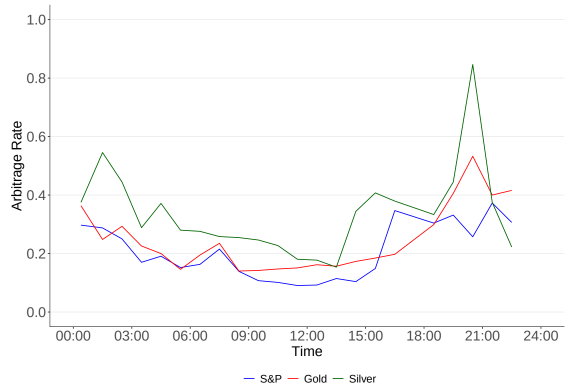
(A) Overvaluation Frequency by Date



(B) Overvaluation Frequency Rates by Date



(C) Overvaluation Frequency Rates by Date



(D) Overvaluation Frequency by Time of Day

FIGURE 4: Overvaluation Frequency by Date and Time

frequencies do reach up to 100%, but this occurs on many days that are roughly uniformly distributed over the sample period.

Figure 4d plots average hourly overvaluation frequency (across all trading days in the sample period). For all three option types, overvaluation frequencies show a noticeable jump around 18:00; this may relate to the fact that these markets resume trading at 18:00 after an hour-long break in the late afternoon. There is thus some evidence that overvaluation frequency may be driven partly by market thinness or price volatility during after-hours trading. But overvaluation frequencies remain around their sample averages (15% for S&P, 19% for gold, and 25% for silver) during traditional trading hours.

We can also address the random price volatility explanation with a placebo test. If the overvaluation anomalies we observe are from random price volatility, then price anomalies should exist just as often in the opposite direction. In other words, our empirical strategy thus far has constructed dominating bull spreads and looked for cases where they cost less than their corresponding binary options, but we could also reverse this logic by constructing *dominated* bull spreads and looking for cases where they cost *more* than their corresponding binary options. Random and symmetric price noise should cause both types of price anomalies to arise at roughly the same rate.

For each binary trade in our sample, we re-run our matching algorithm to look for call option price quotes at strike prices just above the binary strike price, construct a dominated bull spread, and determine how often the dominated bull spread costs more. The results are shown in Table 5: columns 1-3 reproduce our baseline results where the bull spread is dominating, while columns 4-6 report the new placebo results where the bull spread is dominated. Though anomaly frequencies are still nonzero in the placebo test, they are substantially smaller than the baseline results: the S&P anomaly frequency decreases from 15% to 8%, the gold anomaly frequency from 19% to 3%, and the silver anomaly frequency from 25% to 14%. As indicated by the reported t-statistics, the decrease in the anomaly frequency in the placebo test is highly significant (at the 1% level) for all three asset classes. The placebo results indicate that price noise is responsible for some, but not all, of our baseline overvaluation frequency.

5.2 Explicit or Implicit Trading Costs

5.2.1 Collateral Requirements

We also consider the possibility of differential trading costs between the exchanges. As discussed above, we have already accounted for explicit trading fees when identifying cases of overvaluation anomalies. In derivatives markets, collateral requirements are another important source of trading costs, as exchanges often require traders to post collateral (in proportion to the current value or riskiness of their positions) in order to minimize the risk

Table 5: Placebo Test with Dominated Bull Spreads

	Bull Spread is Dominating			Bull Spread is Dominated		
	S&P (1)	Gold (2)	Silver (3)	S&P (4)	Gold (5)	Silver (6)
Anomaly Frequency	0.15	0.19	0.25	0.08	0.03	0.14
t-Statistic for Difference				(29.12)	(32.96)	(8.70)
Binary Trades Considered	39,283	7,434	1,681	39,320	7,443	1,801

Note: Only binary trades that were successfully matched to a bull spread are considered. Columns 1-3 present anomaly measures when the bull spread is constructed to dominate the binary option (the same baseline results shown in Table 3); an anomaly in these cases occurs when the bull spread costs less. Columns 4-6 present overvaluation measures when the bull spread is constructed to be dominated by the binary option; an anomaly in these cases occurs when the bull spread costs more. The t-statistic reported in parentheses tests the hypothesis that the anomaly frequency for each asset class is the same for the dominating-bull-spread and dominated-bull-spread cases.

of counterparty default. But because payoff profiles of a binary option and its dominating bull spread portfolio are always weakly positive (see Figure 1), in neither case can the holder of the position lose more money than the initial trade price. As a result, neither Nadex nor CME requires traders to post collateral when maintaining the positions we consider in this paper.[9] On most major brokerages, traders can obtain clearance to buy bull spreads directly, and our survey in Appendix A.1 directly asks binary option traders whether they have such clearance.

However, for completeness, we perform an additional test to determine whether measures of price volatility for bull spreads predict overvaluation frequency; these would be periods in which, if a brokerage requires collateral to be posted, which they should not - see the discussion in the preceding paragraph - that collateral requirement for bull spreads may increase while the collateral for the binary would remain constant. This may make trading bull spreads implicitly more costly than trading binaries, and if differential collateral requirements are driving our results, we would expect to see anomaly frequency increase as a result (since higher bull spread collateral requirements would deter traders from entering these contracts).

Table 6, which examines the relationship between overvaluation frequency and several

[9]See official exchange material on collateral requirements at Nadex (click for link) and CME (click for link).

Table 6: Overvaluation Frequency and Market Volatility

	Panel A: S&P							
	Same-Day				Lagged			
	Max-Min (1)	P95-P5 (2)	Variance (3)	Hourly Diff. (4)	Max-Min (5)	P95-P5 (6)	Variance (7)	Hourly Diff. (8)
Estimate	0.002	0.003	0.001	0.003	0.002	0.003	0.001	0.138
	(2.475)	(4.246)	(3.933)	(0.398)	(4.720)	(4.394)	(4.301)	(2.930)
R-squared	0.090	0.098	0.080	0.000	0.124	0.087	0.056	0.027
Marginal Effect	0.024	0.025	0.022	0.002	0.028	0.023	0.019	0.013
	Panel B: Gold							
	Same-Day				Lagged			
	Max-Min	P95-P5	Variance	Hourly Diff.	Max-Min	P95-P5	Variance	Hourly Diff.
Estimate	0.002	0.009	0.003	-0.037	0.001	0.005	0.002	-0.599
	(0.825)	(1.687)	(1.459)	(-3.186)	(0.317)	(0.865)	(0.730)	(-1.564)
R-squared	0.002	0.010	0.004	0.010	0.001	0.003	0.002	0.006
Marginal Effect	0.009	0.019	0.012	-0.020	0.005	0.011	0.008	-0.016
	Panel C: Silver							
	Same-Day				Lagged			
	Max-Min	P95-P5	Variance	Hourly Diff.	Max-Min	P95-P5	Variance	Hourly Diff.
Estimate	0.019	-0.721	0.371	-0.563	0.058	-0.460	-0.363	-0.101
	(0.341)	(-1.657)	(0.080)	(-4.199)	(0.695)	(-0.930)	(-0.124)	(-0.339)
R-squared	0.000	0.009	0.000	0.028	0.003	0.003	0.000	0.000
Marginal Effect	0.006	-0.028	0.001	-0.049	0.016	-0.017	-0.001	-0.005

Note: Only binary trades that were successfully matched to a bull spread are considered. The table presents results from simple bivariate regressions where the observations are contract-days, the dependent variable is the contract-specific daily overvaluation frequency, and the independent variable is a measure of price volatility on the CME exchange. The max-min spread is the difference between the day's highest and lowest observed prices, the p95-p5 spread is the difference between the 95th and 5th percentiles of the day's observed prices, variance is computed over all of the day's observed prices, and hourly diff. gives the mean difference between consecutive hourly average prices during the day. Columns 1-4 use volatility measures from the same day on which the overvaluation frequency is computed, while columns 5-8 use volatility measures from the previous trading day. In addition to coefficient estimates, t-statistics (in parentheses), and R-squared values, the table shows the predicted marginal effect on the daily overvaluation frequency of a one-standard deviation increase in the independent variable. Standard errors are clustered by contract, where contracts are defined by their underlying asset (S&P, gold, or silver) and expiration date. CME volatility measures are averaged over all listed strike prices for a contract.

different volatility measures in simple bivariate models, gives some mixed results. While there is little evidence that any of the volatility measures are significantly positively correlated with gold or silver overvaluation frequencies, there is a positive relationship between CME market volatility and S&P anomaly frequency. However, despite the positive coefficient estimates, the R-squared values in the S&P panel peak at 0.12, indicating that most of the variation in daily S&P anomaly frequency remains unexplained.

5.2.2 Liquidity

Market liquidity may be another factor that creates differences in implicit trading costs between the two products. Trading in a thin market is costly, since traders who wish to exit their positions before expiration face a larger bid-ask spread and must pay a larger implicit cost by accepting a less favorable liquidation price. Investors should prefer to trade in more liquid markets, and should be willing to pay a premium (in the form of higher asset prices) in order to do so (Amihud et al., 2005). While we are not able to compare binary and bull bid-ask spreads directly,[10] CME is very clearly the more liquid exchange: many more market makers compete to set prices, many more institutional investors have access, and trading volume is orders of magnitude higher than at Nadex. CME's superior liquidity therefore makes our overvaluation anomaly results even more surprising: investors should be willing to pay a premium in order to trade there, but we find that dominating CME bull spreads often cost less than Nadex binary options. In other words, liquidity considerations actually work against us and make the overvaluation anomalies we detect less likely to occur.

5.2.3 Timing Considerations

We emphasize that bull spreads can be bought directly on standard brokerages (TD Ameritrade, 2021), and therefore the only risk to execution is timing. Most bull spreads are constructed just before the binary option trade, leaving little time for prices to move unfavorably before the binary transaction is completed. Precisely, among the matched trades in our sample, 44% have less than 1 second separating the bull spread and the binary, 60% have a gap of less than 5 seconds, and 74% have a gap of less than 15 seconds; only 12% have a gap greater than 60 seconds.

For completeness, we also compute risks to execution when the bull spread is traded sequentially, though we emphasize that in practice traders may purchase the bull spread directly, without sequential trades (on standard brokerages as TD Ameritrade (2021), and on Nadex by purchasing the call spread). Even in this case, less than 1% of the trades in our sample would be affected, because fewer than that amount feature only one half of the bull spread traded at the same time as the binary. In addition, as discussed prior, for 60%

[10]Our binary option data do not indicate the side of the market on which trades took place, so we cannot infer bid-ask spreads.

of trades, less than 5 seconds separate the bull spread (including both call options used to price it) from the binary. Re-doing our analysis for the subsample where less than 15 seconds separate the bull spread and the binary option shows similar results to the baseline case, with 11% of S&P, 14% of silver, and 20% of gold trades displaying an overvaluation anomaly, with an average overvaluation size/price of 34%, 36%, and 21%, respectively.

5.3 Trade Size

One may imagine binary option traders paying a premium to trade a smaller amount. To control for this, in Table 4 (Panel 'Scale Factor $\leq$ Quantity'), we consider only trades where the observed binary option trade size is at least the size of the comparative bull spread. Even focusing on such trades, overvaluation results persist and are in fact slightly accentuated.

5.4 Barriers to Entry

Investors can open an account and trade directly on Nadex's website, but they can only access CME by opening an account with a third-party broker. These third-party brokers are standard brokerages and are typically easy to access. Nevertheless, to test whether accessibility or knowledge drive our results, we conduct a test *within* Nadex between its own call spreads and binary options. Since both products are offered prominently on the trading screen (see Figure 2) and traders can access both on the same platform, finding overvaluation anomaly results here would show that they persist even after controlling for exchange fixed effects.

Two features distinguish Nadex call spreads from binaries, in addition to payoff structure: first, the price to enter is higher than the price to enter a binary bet; and second, there are far fewer options in terms of strike prices for call spreads than for binaries.[11] To test for

[11] Intuitively, Nadex explains and prices these as if the trader temporarily holds the index, with insurance against extreme movements (though no ownership of the underlying is involved). To illustrate, we provide a hypothetical example. Suppose a binary costs $50 to enter. The binary's maximum net profit and maximum net loss are both $50. Suppose a call spread on Nadex costs $4000 to enter, and the range of payoffs is $3900-$4100 at expiry (which happens within one day). Then the call spread's net profit and loss are $100.

overvaluation, we therefore create a net profit diagram, rather than comparing payoffs and prices separately.

The intuition is shown in Figure 5. For each bull spread traded on Nadex, we find a binary traded near the same time whose positive payoffs are strictly dominated by the positive payoffs of the bull spread (e.g. as in Figure 1). We then test whether the maximum profit:maximum loss ratio of the binary is higher than that of the bull spread. Violations of this condition indicate that the binary and the bull have the relationship depicted in Figure 5: the net profit of the binary is strictly dominated by the net profit of the bull. As a result, under standard theory, the binary should not be traded in equilibrium: any trader who has access to both contracts - as everyone on Nadex does - should always prefer to enter the bull to entering the binary.

Note that if there were a significant gap in time between entry and expiry, then there might be a carry argument for entering the binary over the bull, but all trades we consider here are daily options, which expiry within one day of entry. As a result, carry or interest rate considerations should not matter for this analysis.

Table 7 provides the results of this exercise. There is a traded binary whose net profit is strictly dominated by that of the matched bull spread 15.1% of the time, mirroring the results in our main test. However, relative to the main test, there are fewer trades. Nevertheless, the fact that we obtain similar results within Nadex, where all traders immediately have access to both products, is reassuring.

5.5 Constraints on the Market Maker Side

Delta hedging binary options is more difficult than delta hedging standard options, which may induce market makers to increase the price of the binary (Taleb, 1997); more generally, there may be institutional constraints on the supply side. However, our main finding is not simply that prices are higher on average, but rather a dominance relation. Regardless of the

Thus, net profit and loss profiles are similar, because both price and payoffs are higher for the spread.

Figure 5: Within-Nadex Analysis

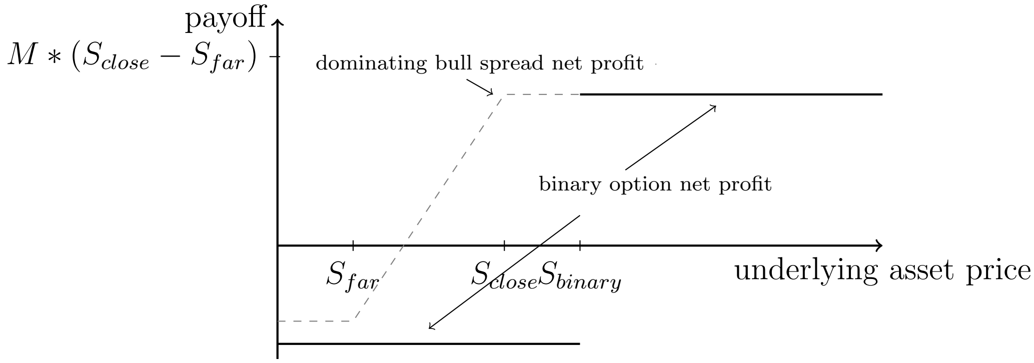


Table 7: Overvaluation of Binaries: Within Nadex

Overvaluation Frequency	0.15
Average Maximum-Profit:Maximum-Loss Difference	1.34
Difference in Trade Time (Minutes)	20.8
Number of Trades Considered	1215

Note: This table considers binary options and call spreads on the S&P 500, all traded within Nadex. Each traded call spread is matched to a traded binary within Nadex, and the trade size is scaled so that the binary makes the same maximum profit as the call spread, but starts making this profit at a strictly higher value of the underlying than the call spread does, making the weakly positive part of the binary state-by-state dominated by the call spread. Since the binary has only two payoffs, to compensate investors for buying the binary whose non-negative payoffs are dominated, the binary should offer a weakly lower loss than the bull spread. Overvaluation anomalies represent those matched trades where the binary's maximum profit:maximum loss ratio is strictly lower than the bull spread's ratio. The second row shows the average difference in ratios conditional on detecting an anomaly.

constraints on the market maker side, under standard theory, a retail investor should never buy a dominated product at a higher price than a dominating alternative. Thus, hedging considerations do not explain buyer behavior in this context.

5.6 Behavioral Explanations

5.6.1 Theories Respecting Dominance

Cumulative prospect theory, and most other well-known revisions to the expected utility model, do not allow for violations of dominance (Kahneman and Tversky, 1992). The overvaluation anomaly result in this case is a violation of dominance. As a result, theories respecting dominance cannot explain it. These also include salience (Bordalo et al., 2013), disappointment aversion (Gul, 1991), and standard expected utility (Von Neumann and Morgenstern, 2007).

5.6.2 Prospect Theory

While original prospect theory (Kahneman and Tversky, 1979) allows for violations of dominance, it does not accord with our empirical findings; prospect theory agents would never choose a dominated binary option over a dominant bull spread. The reason is that the way in which prospect theory allows for dominance violations is by distorting probabilities. However, in a state-by-state dominance relationship, no matter how probabilities are distorted, the dominating bull spread still looks more appealing, because it provides a higher payoff in every state. We provide a formal mathematical proof next.

Proposition 1. *Given two derivatives O and B, where O state-by-state dominates B, original prospect theory predicts that the willingness to pay for O should be higher than the willingness to pay for B, regardless of the probability weighting function, reference point, or utility function used.*

Proof. Observe that for any probability weighting function π and reference point R, the utility a prospect theory agent receives from a binary options or bull spread is

$$U = \int \pi(p(s))u_R(q(s))ds, \tag{7}$$

where s is the underlying asset price at contract expiration, u_R is a utility function with the R subscript denoting dependence on the reference point, $p(s)$ is the probability of underlying asset price s occurring, and $q(s)$ is the monetary payoff that the agent receives from the option portfolio when underlying asset price s occurs. Consider a prospect theory agent who buys a binary option with strike price K. Since the binary option pays \$100 if $s > K$ and zero otherwise, the utility of the binary option is

$$U(B) = \int_0^K \pi(p(s))u_R(0)ds + \int_K^\infty \pi(p(s))u_R(100)ds. \tag{8}$$

In contrast, the comparable bull spread O in our empirical test (constructed by buying a call option with strike $K - \epsilon$ and selling a call option with strike K) gives utility[12]

$$U(O) = \int_0^{K-\epsilon} \pi(p(s))u_R(0)ds + \int_{K-\epsilon}^K \pi(p(s))u_R(s-(K-\epsilon))ds + \int_K^\infty \pi(p(s))u_R(100)ds. \tag{9}$$

Because $\pi(\cdot)$ is weakly positive and $u_R(\cdot)$ weakly increasing in prospect theory, it follows that, for any $\epsilon > 0$,

$$\begin{aligned}\int_0^{K-\epsilon} \pi(p(s))u_R(0)ds + \int_{K-\epsilon}^K \pi(p(s))u_R(s-(K-\epsilon))ds & \\ \geq \int_0^{K-\epsilon} \pi(p(s))u_R(0)ds + \int_{K-\epsilon}^K \pi(p(s))u_R(0)ds & \\ = \int_0^K \pi(p(s))u_R(0)ds, & \end{aligned} \tag{10}$$

and therefore that $U(O) \geq U(B)$ for any prospect theory agent, regardless of functional

[12]For simplicity but without loss of generality, we assume that the notional value of the call options is such that the portfolio pays exactly 100 if the underlying asset price is above K at expiration.

form. This shows that an original prospect theory agent cannot strictly prefer a dominated binary option to a comparable bull spread, and thus that prospect theory cannot explain overvaluation of 0-1 bets as documented here. □

5.6.3 Ambiguity Aversion

Under subjective expected utility (Savage, 1972), the agent assigns subjective probability distribution to states of the world, which here are various underlying asset prices. However, in our test the bull spread O pays out weakly more money than the binary option B in all states of the world, and consequently for any probability distribution over underlying asset prices, it remains the case that O should be weakly more attractive than B.

Max-min preferences (Gilboa and Schmeidler, 1989) and robust control (Hansen and Sargent, 2001) have the agent behave as though he were maximizing utility against adversarial nature, which can choose the worst possible probability distribution for the action chosen (subject to model-dependent constraints). However, since O pays out weakly more money than the binary option B in all states of the world, for any given probability distribution over underlying asset prices, O is weakly more attractive than B; and in particular, this is also the case for the worst possible probability distribution. Consequently, these theories also predict that the price of O should be weakly higher than the price of B.

We provide a formal proof for the robust control formulation (Hansen and Sargent, 2001), which is the form of ambiguity aversion typically used in asset pricing (Drechsler, 2013; Ilut and Schneider, 2022). The intuition for max-min preferences is similar- no probability distortion changes the fact that the bull spread pays more in every state.

Proposition 2. *Given two derivatives O and B, where O state-by-state dominates B, robust control predicts that the willingness to pay for O should be higher than the willingness to pay for B for any strictly increasing Bernoulli utility.*

Proof. Let d be a derivative mapping states (e.g. values of the underlying asset) to payoffs. The utility of derivative d under robust control is given by: $U(d) = \min_{p\in\Delta S} \int u(d)dp +$

$\theta R(p||q)$, where ΔS is the set of possible subjective probability distributions, u is the Bernoulli utility, q is a best-guess probability distribution, and the function R denotes the relative entropy.

Since O state-by-state dominates B, it also first order stochastically dominates B for any distribution p, e.g. $\int u(O)dp > \int u(B)dp$ for all strictly increasing functions u. As a result,

$$U(O) = \min_{p\in\Delta S} \int u(O)dp + \theta R(p||q) > \min_{p\in\Delta S} \int u(B)dp + \theta R(p||q) = U(B),$$

completing the proof.

□

5.6.4 Gambling

Past work hypothesizes that some investors may like to gamble (Markowitz, 1952) or have a preference for lottery-like stocks arising from prospect theory (Barberis and Huang, 2008). Liking gambling alone would not explain our results because one can gamble with both products, and the gambling explanation does not predict a liking for one product over another. We prove above that prospect theory, and therefore a preference for lottery-like investments arising from it, cannot explain our results.

5.6.5 Duration-Based Risk Aversion

Prior work (van Binsbergen et al., 2012; Lazarus, 2022) finds that investors' measured risk aversion can differ by expiry horizon. Here, however, we match the binary option and bull spread on expiry, so there is no duration-based difference between them.

5.6.6 Rational Inattention

As discussed earlier, investors know about, and have access to, both binary options and bull spreads. However, it may be the case that they pay more attention to binaries than to bull spreads.

If we were to try to capture this idea with a standard rational inattention model, per Matějka and McKay (2015), the probability of selecting an asset, say choosing to buy a binary versus a bull spread, is given by:

$$P(a_i|v) = \frac{P^0_{a_i} e^{v_{a_i}/\lambda}}{\sum_{a_j} P^0_{a_j} e^{v_{a_j}/\lambda}},$$

where a is an asset, v is the state of the world (for example, the current value of the underlying), v_a is the asset's payoff, and P is the prior probability of selecting the asset.

This equation tells us that, under rational inattention, the probability of paying attention to an asset is proportional to the asset's payoff. By design, however, the bull spreads we consider have strictly higher payoffs than the binary options. Rational inattention would therefore predict more attention is paid to the better product, the bull spread, which is not in line with what we find in our data.

5.7 One Consistent Explanation

We are not attached to any particular explanation for our results, beyond noting that standard preference models, including Nobel-prize winning theories, cannot capture them.

For completeness, we offer one consistent explanation for overvaluation of 0-1 bets; other explanations may be equally valid. It is possible that the anomaly arises from cognitive constraints because the binary is easier to understand, in a number of ways, than the bull spread. First, it has only two outcomes, making computation of its expected value or other moments arguably easier than the multi-leg bull spread. Second, it may be linguistically easier to explain - a yes-no bet requires only a few words to conceptualize; the bull spread, even when described intuitively as Nadex does (Nadex, 2024), requires some more explanation. Third, the binary has fewer kinks in its payoff and profit functions than the bull spread, which may make mentally visualizing it easier.

Our findings are thus in line with, and provide one real-world impetus for, a newer direction in behavioral finance, that of cognitive models and constraints as drivers of behavior (ex. McDonald and Rietz (2018); Gödker et al. (2023); Barberis and Jin (2023); Puri (2024); Fedyk et al. (2024)).

6 Conclusion

In this empirical paper, we document a new anomaly in a unstudied retail market which provides a clean setting for preference tests. Trades in this market overvalue 0-1 bets about one fifth of the time regardless of underlying asset considered, both within and across platform. We show that our results cannot be captured by prospect theory or ambiguity aversion regardless of functional form used, nor by other popular behavioral preference models. We also test for myriad financial justifications and show that none fully account for this overvaluation. Our results are consistent with a more cognitive explanation, since binaries are intuitively easier to understand than option spreads.

Whereas work on utility models post prospect theory is typically in the laboratory with small stakes, our retail market provides a high-stakes, real-world, setting for testing prominent theories.

Our work informs models of retail behavior by documenting a new anomaly. It also provides a data point that future behavioral work can use as a building block for developing new theories. For example, any cognitive process that can microfound overvaluation of 0-1 bets would, because of our theoretical results, immediately have observational value-add relative to the most prominent utility frameworks.

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A Appendix: Additional Supplemental Tests

In this section we perform two additional supplemental tests. Neither test is critical for our results; they are meant to add additional detail for curious readers.

First, we run a survey of binary option traders to obtain their demographics and trading behavior. The median binary option respondent is young (20-40), male, white, college-educated, employed, makes 100-150k per year, has been trading binary options for over two years, buys rather than sells binaries, and has traded bull spreads in the past.

Second, for completeness, we obtain theoretical Black-Scholes predictions for the prices of in-the-money binaries, and show that binaries are overpriced relative to this standard theory. Our main test in Section 4 is more demanding because it requires dominance violations, while the Black-Scholes test shows deviations from a theoretical benchmark. Unsurprisingly because it is a weaker test, the Black-Scholes test yields a higher percentage of overvaluation results, implying that in-the-money binaries are overvalued two thirds of the time.

A.1 Survey

To better understand the profile, experience, and preferences of binary option traders, we run a survey of traders by posting on an online binary options discussion forum. Survey methodology details are in Online Appendix B.3. The survey yielded a final sample size of 127 respondents.

Table A.1 reports respondents' demographic details, respondents' reported access to CME brokerages, supplemented by a follow-up survey. As discussed in Section 5.4, a further robustness check compares bull spreads to binaries within Nadex, where we know all participants have access to both products.

Survey respondents were also asked to answer incentivized lottery choice questions, two of which directed tested prospect theory and one of which tested for dominance violations. Only 14% of respondents answered in a manner consistent with prospect theory, and 60%

Table A.1: Binary Option Traders Survey

Gender		**Race**	
Male	67%	White	79%
Female	33%	Black	15%
		Other	6%
Age		**Trading Experience**	
20-29	35%	$<$ 1 year	21%
30-39	59%	1-2 years	25%
40-49	5%	$>$ 2 years	54%
50-59	1%		
Education		**Household Income**	
Some college	9%	$<$ \$50,000	2%
2-Year degree	33%	\$50,000-\$99,999	28%
4-Year degree	43%	\$100,000-\$149,999	33%
Master's degree	13%	\$150,000-\$199,999	20%
Doctoral degree	1%	\$200,000-\$249,999	11%
		\$250,000-\$299,999	5%
		\$300,000+	1%
Employment		**Access**	
Paid employee	71%	Has CME trading account	98%
Self-employed	22%	Traded calls or bulls within two years	78%
Not working	7%	Able to trade spreads or short calls on their brokerage account	70%
Share of Financial Wealth in Risky Assets		**Share of Risky Assets in Binary Options**	
0%-20%	3%	0%-20%	7%
20%-40%	28%	20%-40%	35%
40%-60%	40%	40%-60%	37%
60%-80%	27%	60%-80%	17%
80%-100%	2%	80%-100%	3%

Note: This table reports the results of a survey of binary option traders, posted on a binary option trading discussion forum. Within the knowledge category, the final two rows come from a follow-up survey. Additional details can be found in Online Appendix B.3.

violated dominance towards a lower-outcome lottery.

A.2 Black-Scholes

Our preferred comparison for the price of the binary options are the dominating bull spreads analyzed so far, because they are tradeable products. However, these real-world products are 'much more' dominating than the binary option, in the sense of Figure 1: the strike prices cannot be made arbitrarily close to that of the binary option, and only integer contracts can be traded, so that the dominating bull spread pays strictly much more in all states of the world, rather than strictly more in some states and weakly more in others. This means our primary test for mispricing is quite demanding: and it is possible that, at times, these very dominating bull spreads are not more expensive than the binary option, but that a less dominating bull spread (that still dominates the binary) would be. Our analysis above is therefore a conservative estimate of the level of mispricing in the binary option market.

An alternative test would be to calculate an ideal market-based price for the binary option, and compare its traded price to this ideal. In this section, we calculate such an ideal price for S&P 500 weekly binaries using the canonical Black-Scholes-Merton (BSM) pricing formula for binary options (James, 2003; Black and Scholes, 1973; Merton, 1973):

$$\text{Black-Scholes-Merton (Binary)} = e^{-r(T-t)}N(d),$$
$$\text{where } d = \frac{ln\frac{S_t}{K} + (r - q - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}},$$

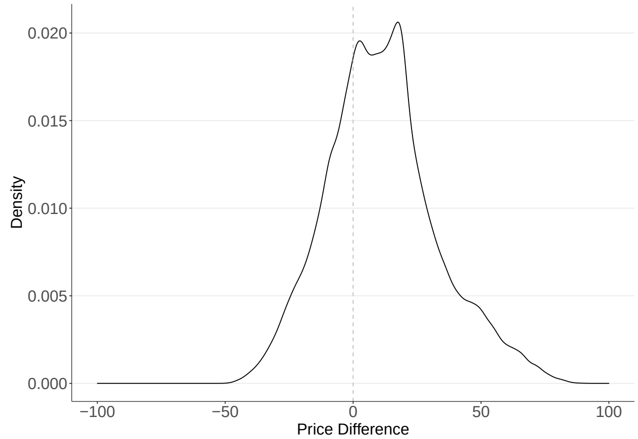
and r is the risk-free rate, $T-t$ is the time to expiry, N is the CDF of a standard normal, S_t is the price of the underlying at the time of purchase, K the strike price, q dividends, and σ^2 the volatility of the underlying.

BSM makes several modeling assumptions that more involved non-parametric approaches (for example, Breeden and Litzenberger (1978)) dispense with, but prior literature finds the canonical BSM approach to have similar to better performance on in-the-money index options (Gradojevic and Kukolj, 2022; McKenzie et al., 2007; Bennell and Sutcliffe, 2004).[13] Given this literature, the parsimony of the BSM model, and its continued use as a benchmark, we focus here on BSM predictions for in-the-money options on S&P 500 futures only, though acknowledge that, on other products, approaches other than BSM may be preferred.

To map the model to the data, the strike price and days to expiry are inherent to the binary option; the price of the underlying is the S&P futures price (Nasdaq, 2022); the risk free rate is the three-month treasury rate (Board of Governors, 2022*a*); and the expected volatility of the underlying is given by the VIX index (Board of Governors, 2022*b*).

[13] For example, Gradojevic and Kukolj (2022) writes: 'one of the deepest and the most liquid option markets in the United States, the S&P 500 index option market is sufficiently close to the theoretical setting of the Black-Scholes model.'

Figure A.1: Differences Between In-the-Money Binary Option Prices and Black-Scholes Valuations



Note: Positive values indicate trade prices that overvalue binary options relative to their Black-Scholes valuations.

For a conservative approach, we set dividends $q = 0$. To see why this is conservative, observe that the BSM price of the binary option decreases in dividend. Setting $q = 0$ is therefore an upper bound for the BSM price. Because our test is for whether the traded price of the binary option exceeds the BSM valuation, setting an upper bound on the BSM valuation under-estimates the frequency with which the binary option is more expensive than it should be under this canonical framework.

Of the in-the-money S&P 500 binary option trades in our dataset, 66% are priced higher than their BSM valuation. Figure A.1 shows the distribution of price differences, confirming traded prices are on average higher than BSM valuations, in a statistically significant way ($p < 0.01$). The average price difference between the traded and theoretically predicted price is \$11.30, which corresponds to traded prices being 28% higher than theoretically predictions.